

Extrapolated UT1 Effect on VLBI Clock Sync

J. W. Layland and B. V. H. Saxberg
Communications Systems Research Section

In this article we calculate the two-sample Allen Variance of the extrapolated UT1 numbers, and use this calculation to infer its effect on DSN clock stability measures obtained via VLBI clock synchronization. For measurement time $T < 300$ s, or $T > 3$ days, the error in UT1 and its variations do not seriously degrade the stability measurement relative to anticipated variations in the clocks themselves.

I. Introduction

The DSN is currently beginning the attempt to monitor the stability of its clocks and frequency standards in the 64-meter net by means of VLBI. The utility of this monitoring effect depends upon both its accuracy and the timeliness of the measurement results.

Many recent VLBI clock sync experiment data sets have been received and processed within less than a week of the acquisition of this data. Software which processes this data must know the Earth's precise (angular) position (i.e., UT1) within the Quasar coordinate frame. In Ref. 1, it was assumed that this positional information would be available in the form of the BIH "Rapid Service" measurements of UT1, the Earth's rotational phase, and polar motion (Ref. 2). For real-time or near-real-time processing, this form of the data is not available, and we must either rely upon extrapolated values for UT1, or obtain data in a form which can be used to solve for UT1. The fundamental purpose of the VLBI clock sync experiments is to measure the relative stability of the frequency standards in the three 64-m tracking stations of the DSN. This stability (instability) is usually specified in terms of the square-root of

the two-sample Allan Variance (Ref. 3) of the time differences of two clocks. We calculated this instability measure for UT1 itself, in its several available forms, and used this result to infer its effect on VLBI Clock Sync.

II. Inter-Day Instability of UT1 Values

The data set analyzed consisted of the USNO Extrapolated, BIH "Rapid Service," and BIH "Final" Values for UT1-UTC for the period 76 July 1 to 78 May 1, taken from the USNO Time and Frequency Bulletins, Series 7. Figure 1 shows log-log plots of the square-root Allan Variance for these three data types. Both the extrapolated and rapid service UT1 have regions of high variance in the smaller measurement times, due presumably to white-noise effects upon the raw observations from which UT1 is derived. The final UT1, being derived with the help of observations in a several-week time span surrounding each day for which it is computed, shows the effect of this smoothing with a much decreased variance in this region. All three, extrapolated, rapid-service, and final value UT1, reach a peak at a roughly 60- to 90-day measurement

interval. This peak presumably results from seasonal variations in the Earth's rotation, which are known to exist but hard to predict. The amplitude of this peak is itself seasonally variable when the Allan Variance is computed for six-month time intervals (not shown). The dramatic dropoff for measurement intervals of over 200 days is an artifact of the short time span of data analyzed since it disappears over a five-year interval (Ref. 1, Fig. 3).

Figure 2 shows log-log plots of the square-root Allan Variance of the pair-wise differences between these three data types. All three curves show the effect of white-noise for measurement intervals less than five days. This noise adds as if independent between the extrapolated and rapid-service UT1 values, which further reinforces the identification of this segment as white (independent) noise of measurement. The curves using extrapolated data reach a peak at 30- to 60-day measurement times, indicating that the seasonal variations are not well predicted in the extrapolation process. The rapid service-final curve has a small bump in this region, but for the most part, the variational patterns in these two estimates of UT1 agree increasingly well as the measurement intervals increase. The interesting kink which appears in all these curves at the 150- to 200-day measurement interval is probably an artifact of the small data set used.

Errors in UT1 can masquerade as clock errors in VLBI time delay measurements to the tune of at worst $2 \mu\text{s}$ clock error per second of UT1 error. When an inter-day Allan Variance of these clocks is derived from the VLBI clock offsets, the error in UT1 contributes a term to the square-root Allan Variance which is a factor 2×10^{-6} smaller than the square-root Allan Variance of the error in the estimate of UT1 itself. If we assume that the BIH Final Values follow correctly all of the (slow) variations in UT1, then the square-root Allan Variance of the differences plotted in Fig. 2 closely represents the instability measure of the error in following UT1 by the extrapolated (or rapid service) estimate of it. For $t > 3$ days, these numbers are all less than 5×10^{-9} , and hence their contribution to the clock instability measure is less than 10^{-14} . Thus it seems that extrapolated UT1 will contribute to the clock instability measure roughly at the level of the H-masers themselves, while rapid service UT1 would contribute negligibly for $t > 10$ days.

III. Short-Term Effects

The VLBI-observed frequency shift can also be used to estimate the short-term stability of the interferometer, including, of course, the two frequency standards. In particular, the absolute difference of the interferometer frequency shift measured over two consecutive T -second time intervals is one sample of the square-root Allan Variance. This measure, too, is affected by errors in Quasar position and UT1. We have no information on the very short-term variations of UT1 from the BIH data because they are masked by observing errors in the BIH raw data. We expect these short-term variations to be small, presumably less than the 2×10^{-9} observed at 30 to 300 days, and that their effect on the measured $\sigma(\Delta F/F)$ would be dominated by the relatively constant error in the daily estimate of UT1.

The VLBI fringe frequency equations (Ref. 4) can be easily manipulated to show the effect of a constant error in UT1 on the measured stability. For the frequency measurement interval T , and a fixed error E_{UT}

$$\sigma|_T(\Delta f/f) = \frac{\omega e}{\sqrt{2}} \frac{r_b}{c} \cdot (\omega e \cdot E_{UT}) \cdot (\omega e \cdot T) \\ + \text{other effects}$$

where ωe is the Earth rotation rate in radians/s, r_b is the length of the interstation baseline projection onto the equatorial plane, and c is the speed-of-light.

IV. Conclusion

Figure 3 shows the anticipated effect of fixed errors in UT1 upon the VLBI frequency stability measurements, for UT1 errors of 3 ms (Rapid Service) and 10 ms (extrapolated, conservative) on the DSN intercontinental baselines. The effect of UT1 errors on inter-day time stability measurement is also shown. For measurement time $T < 300$ s or $T > 3$ days, the error in UT1 and its variations do not seriously degrade the stability measurements. For the region between, the UT1 effect via either time or frequency measurements is large enough to mask the variations in the frequency standards unless UT1 is concurrently solved for.

References

1. Layland, J. W., and Mathews, A. N., "VLBI Clock Sync and the Earth's Rotational Instability," in *The Deep Space Network Progress Report 42-42*, pp. 81-84, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 1977.
2. Feissel, Martine, "The Determination of UT1 by the Bureau Internationale de l'Heure," Proceedings of the Eighth Annual Precise Time and Time Interval Meeting, pp. 47-54, USNO, Dec. 1976, Washington, D.C.
3. Barnes, J. A. et al., Characterization of Frequency Stability, NBS Technical Note 394, Oct. 1970.
4. Williams, J. G., "Very Long Baseline Interferometry and Its Sensitivity to Geophysical and Astronomical Effects," *Space Programs Summary 37-62*, Vol. II, pp. 49-55, Jet Propulsion Laboratory, Pasadena, Calif., Mar. 1970.

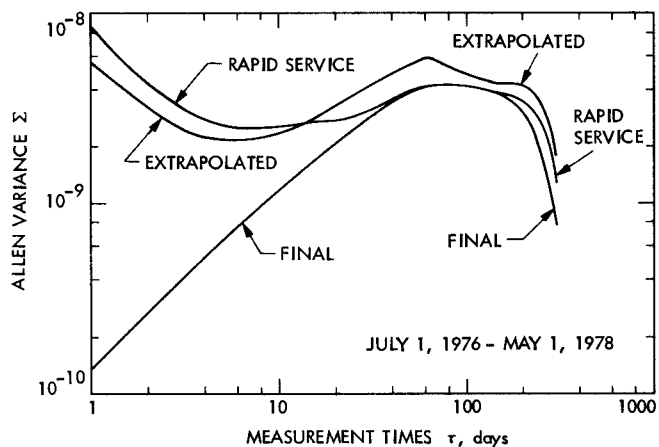


Fig. 1. Square-root Allan Variance of Earth rotation data (UT1)

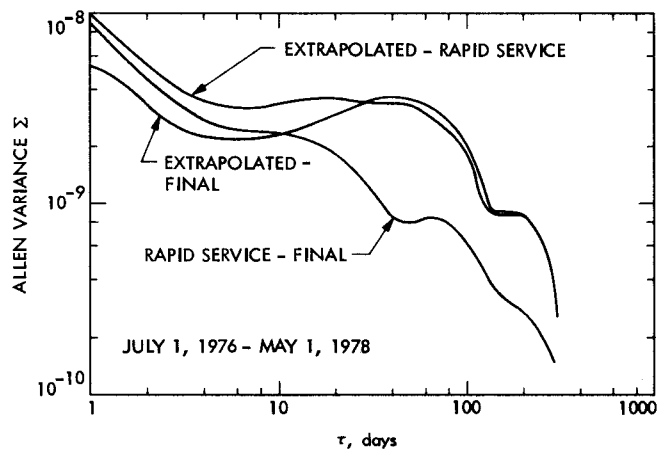


Fig. 2. Square-root Allan Variance of pair-wise differences between various UT1 estimates

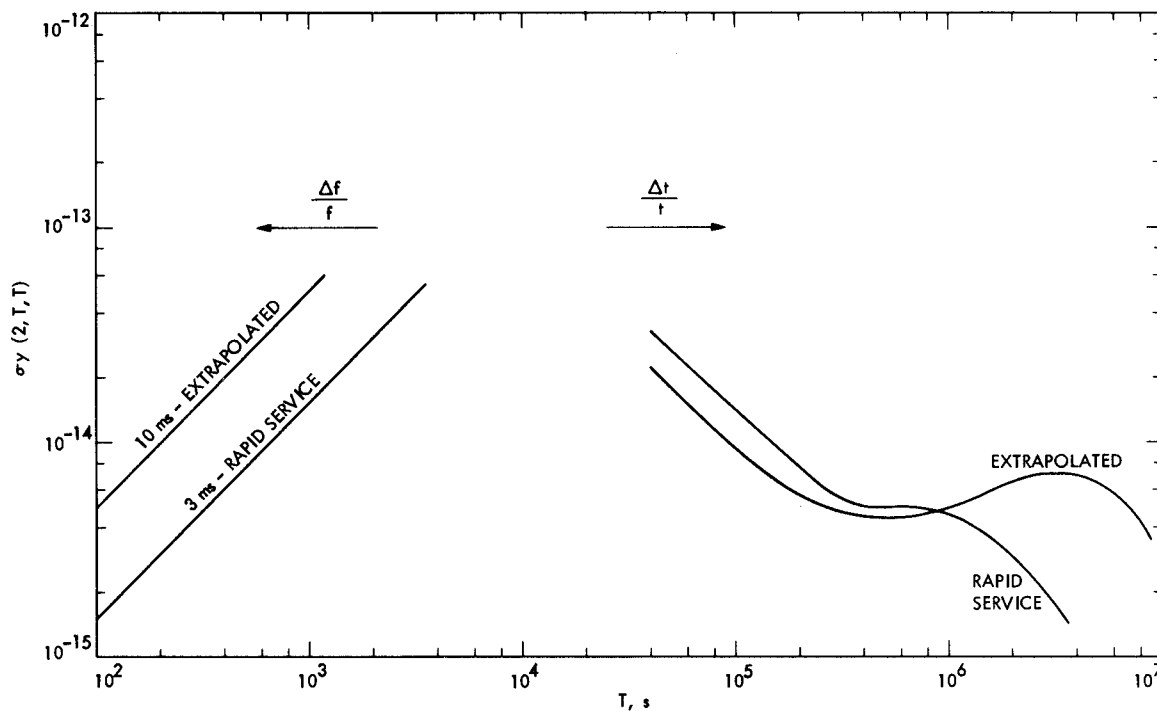


Fig. 3. Anticipated UT1 effect on VLBI clock-stability measurements